Coefficient Inequality and Coefficient Bounds for a New Subclass of Bazilevic Functions

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Received: 09/09/2023	**Accepted:27/09/2023**

ملخص:

في هذا البحث، قمنا بتقديم فئة فرعية معممة جديدة من دوال بازيليفيتش، وهي فئة معينة من الدوال التحليلية والاحادية المحددة في قرص الوحدة المفتوحة. ثم نقوم بدراسة متباينة المعاملات و المعاملات المحدودة لهذه الفئة الفرعية. وتحصلنا على العديد من النتائج لمؤلفين سابقيين.

Abstract

In this paper, the researcher introduced a new generalized subclass of Bazilevic functions, which are a particular class of analytic and one to one functions defined in the open unit disc. Then a study coefficient inequality and coefficient bounds for this subclass was performed. As a result, several dervations for previous authers was obtained.

Keywords: Analyticfunctions,Bazilevic funcions, coefficient inequality, coefficient bounds, Generalization derivative operator.

Introduction

The Bazilevic function is a type of univalent function, which is an analytic and one-to-onefunction in the unit disc. It plays an essential role in the field of complex analysis. A subclass of Bazilevic functions would refer to a particular set of these functions that share certain additional properties or characteristics. These subclasses can be defined based on various criteria, such as the behavior of the function in certain regions, the values of their coefficients, or their relationship to other classes of functions**.** For the importance of the class Bazilevic Functions , many authors studied these types of the subclass of Bazilevic functions. For instance, (Kim,2009) investigated the growth theorem of Bazilevic functions of type (α, β), also (Oladipo & Olatunji, 2010) studied some of the properties of certain subclass of Bazilevic function defined by Catas operater.as well as, (Arif. et al, 2011) introduced the new class of strongly Bazilevic functions by using a generalized Robertson function and give some interesting properties of this class. In addition, (Amer&Dures,2012) studied distortion theorem for class of Bazilevic Functions. Furthermore, (Amer. et al, 2018) defined a subclass of uniformly Bazilevic Functions using new generalized derivative operator. Recently (Breaz**.**et al., 2022) introduced a new class of Bazilevic functions involving the Srivastava-Tomovski generalization of the Mittag-Leffler function and they obtained coefficient estimates,subordination conditions for starlikeness and Fekete–Szegö functional. Despite, the amount of previous researches that focused on this type of functions. On the other hand, there are still a lot of interest about propriety of Bazilevic functions, that lead us as authors for this paper to study coefficient inequality and cofficient bound for the new subclass of Bazilevic functions which is defined by a generalized derivative operator $D^{\alpha,\delta}(m,q,\lambda)$.

Let $U=\{z \in C:|z|<1\}$, be the unit disc in the complex plane, and let A be the class of functions which are anaylyticand normalized by the condition $f(0) = 0, f'(0) = 1$ in U .It has a Taylor series representation

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad (z \in U), \quad (1).$$

The class P consists of all functions of the form

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots + c_k z^k = 1 + \sum_{k=1}^{\infty} c_k z^k, \quad (z \in U),$$

that are analytic in U such that $p(0) = 1$ and $\Re\{p(z)\} > 0, z \in U$.A function f in P is called a function with positive real part in U.

1. Preliminaries

The authors in [1,2] introduced a generalization derivative operator $D^{\alpha,\delta}(m,q,\lambda)$, as the following:

$$D^{\alpha,\delta}(m,q,\lambda) = z + \sum_{k=2}^{\infty} k^{\alpha}(1+\frac{k-1}{1+q}\lambda)^m c(\delta,k) a_k z^k, \tag{2}$$

where $k,\delta,\alpha \in N_0 = \{0,1,2...\}, m \in Z,\ \lambda,q \geq 0,$ and $c(\delta,k) = \frac{(\delta+1)_{k-1}}{(1)_{k-1}}$.

Using the operator above we give the definition of a more larger and generalized subclass of Bazilevic functions as follows:

Definition 1.2 *Let $T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$ denote the subclass of A consisting of functions f which satisfy the inequality*

$$\Re\left\{\frac{D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)}{(\frac{1+\lambda(\beta-1)+q}{1+q})^m z^{\beta}}\right\} > \gamma,$$

where $\lambda,q \geq 0, \beta > 0 \quad (\beta \quad is \quad real)$ and $\alpha,\delta \in N_0, m \in Z, 0 \leq \gamma < 1$.

Base on Definition 1.2 above, we have the following remark to make.

Remark

1) For $\alpha = \delta = 0, q = 0$ and $m \in N_0$, we have

$$\Re\left\{\frac{D^{0,0}(m,0,\lambda)f^{\beta}(z)}{(1+\lambda(\beta-1))^m z^{\beta}}\right\} > \gamma \equiv \Re\left\{\frac{D_{\lambda}^{m} f^{\beta}(z)}{(1+\lambda(\beta-1)^m z^{\beta}}\right\} > \gamma,$$

where D_{λ}^{m} is the Al-Oboudi derivative operator.While this class is studied in [3].

2) For $\alpha=\delta=0, q=0, \lambda=1$ and $m \in \mathrm{N}_0$, we have

$$\Re\left\{\frac{D^{0,0}(m,0,1)f^{\beta}(z)}{\beta^m z^{\beta}}\right\}>\gamma \equiv \Re\left\{\frac{D^m f^{\beta}(z)}{\beta^m z^{\beta}}\right\}>\gamma,$$

where D^m is the S$\hat{a}$l$\hat{a}$gean derivative operator. While this class is studied in [3].

3) For $\beta=1, \alpha=\delta=0, \gamma=0, m=0$ we have

$$\Re\left\{\frac{D^{0,0}(0,q,\lambda)f(z)}{z}\right\}>0 \equiv \Re\left\{\frac{f(z)}{z}\right\}>0,$$

which is the class of functions studied in [4].

For the purpose of simplicity and clarity we wish to state the following function, from (1) we can write that.

$$\left(f(z)\right)^{\beta}=\left(z+\sum_{k=2}^{\infty}a_k z^k\right)^{\beta}.$$

Using binomial expansion we have

$$\left(f(z)\right)^{\beta}=z^{\beta}+\sum_{k=2}^{\infty}a_k(\beta)z^{\beta+k-1}, (3)$$

where the coefficients a_k shall depend so much on the parameter β.
Applyingeq (3) in derivative operator (2), we obtain

$$D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)=\left(\frac{1+q+\lambda(\beta-1)}{1+q}\right)^m z^{\beta}+$$

$$\sum_{k=2}^{\infty}k^{\alpha}\left(\frac{1+q+\lambda[\beta+k-2]}{1+q}\right)^m c(\delta,k)a_k(\beta)z^{\beta+k-1}.$$

In order to derive our main results, we have to recall here the following lemma:

Lemma 1.3[8] A function $p \in P$ satisfies $\Re\{p(z)\}>0, (z \in \mathrm{U})$ if and only if

$$p(z) \neq \frac{(\Psi-1)}{(\Psi+1)} \quad (z \in U, |\Psi|=1)$$

2 Coefficient inequality for functions in the subclass $T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$

We intend to derive the following theroem for the purpose of our next result.
Theorem2.1 *A function $f \in A$ is in the class $T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$ if and only if*

$$1+\sum_{k=2}^{\infty}A_k z^{k-1} \neq 0,$$

where

$$A_k=\frac{(\psi+1)}{2(1-\gamma)}k^{\alpha}\left(\frac{1+q+\lambda[\beta+k-2]}{1+\lambda(\beta-1)+q}\right)^m c(\delta,k)a_k(\beta).$$

Proof: Upon setting

$$p(z)=\frac{\dfrac{D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)}{(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}}-\gamma}{1-\gamma},$$

for $f(z)\in T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$,we obtain that $p(z)\in P$, and $\Re\{p(z)\}>0, z\in U$.
Using Lemma 1.3, we have that

$$\frac{\dfrac{D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)}{(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}}-\gamma}{1-\gamma}\neq\frac{\psi-1}{\psi+1},\quad (z\in \mathrm{U}),\quad for\quad all\quad |\psi|=1.$$

Then ,

$$(\psi+1)\left[D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)-\gamma(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}\right]\neq(\psi-1)(1-\gamma)(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta},$$

which readily yields

$$(\psi+1)D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)+(1-2\gamma+\psi)(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}\neq 0.$$

Thus we find that

$$(\psi+1)\left(\frac{1+q+\lambda(\beta-1)}{1+q}\right)^{m}z^{\beta}+(\psi+1)\sum_{k=2}^{\infty}k^{\alpha}\left(\frac{1+q+\lambda[\beta+k-2]}{1+q}\right)^{m}c(\delta,k)a_{k}(\beta)z^{\beta+k-1}+$$

$$(1-2\gamma-\psi)(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}\neq 0,$$

that is

$$(\psi+1)\sum_{k=2}^{\infty}k^{\alpha}\left(\frac{1+q+\lambda[\beta+k-2]}{1+q}\right)^{m}c(\delta,k)a_{k}(\beta)z^{\beta+k-1}+2(1-\gamma)(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}\neq 0. (4)$$

Dividing the both sides of (4) by $2(1-\gamma)(\frac{1+\lambda(\beta-1)+q}{1+q})^{m}z^{\beta}$.

$$1+\sum_{k=2}^{\infty}\frac{(\psi+1)}{2(1-\gamma)}k^{\alpha}\left(\frac{1+q+\lambda[\beta+k-2]}{1+\lambda(\beta-1)+q}\right)^{m}c(\delta,k)a_{k}(\beta)z^{k-1}\neq 0,$$

which completes the proof .

Setting $q=l, m=n, \alpha=\delta=0$ in Theorem2.1, we get result in[6].

Corollary2.2 *A function $f(z)\in A$ is in the class $\mathrm{T}^{0,0}(\mathrm{n},\mathrm{l},\lambda,\beta,\gamma)\cong \mathrm{T}_{\mathrm{n}}^{\beta}(\mathrm{l},\lambda,\gamma)$ if and only if*

$$1+\sum_{k=2}^{\infty}A_k z^{k-1}\neq 0,$$

where

$$A_k=\frac{\psi+1}{2(1-\gamma)}\left(\frac{1+\lambda[\beta+k-2]+l}{1+\lambda(\beta-1)+l}\right)^n a_k(\beta).$$

Setting, $q=0, m=\gamma=\alpha=\delta=0, \beta=1$ in Theorem 2.1, we get result in[5].

Corollary 2.3 *A function $f(z)\in A$ is in the class $T^{0,0}(0,0,\lambda,0,1)\cong T(\alpha)$ if and only if*

$$1+\sum_{k=2}^{\infty}A_k z^{k-1}\neq 0,$$

where

$$A_k=\frac{\psi+1}{2(1-\beta)}a_k.$$

Theorem 2.4 *If $f(z)\in A$ satisfies the following condition:*

$$\sum_{k=2}^{\infty}\left|\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j}j^{\alpha}(1+\lambda(\beta+j-2)+q)c(\delta,j)a_j(\beta)\begin{pmatrix}\mu\\ t-j\end{pmatrix}\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right|$$

$$\leq(1-\gamma)(1+\lambda(\beta-1)+q),$$

Where $\lambda,q\geq 0,\beta>0$ $(\beta$ *is real*$)$ and $\alpha,\delta\in\mathrm{N}_0, m\in\mathrm{Z}, 0\leq\gamma<1.$
$\nu,\mu\in\mathrm{R}$ and then $f(z)\in T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma).$

Proof:

First of all, we note that $(1-z)^{\mu}\neq 0, (1+z)^{\nu}\neq 0$ $\nu,\mu\in\mathrm{R}, z\in\mathrm{U}.$

Thus to prove

$$1+\sum_{k=2}^{\infty}A_k z^{k-1}\neq 0.$$

Hence, if the following inequality

$$(1+\sum_{n=2}^{\infty}A_k z^{k-1})(1-z)^{\mu}(1+z)^{\nu}\neq 0, \tag{5}$$

holds true, then we have

$$1+\sum_{k=2}^{\infty}A_k z^{k-1}\neq 0,$$

It is easily seen that (5) is equivalent to

$$\left(1+\sum_{k=2}^{\infty} A_k z^{k-1}\right)\left(\sum_{k=0}^{\infty}(-1)^k b_k z^k\right)\left(\sum_{k=0}^{\infty} c_k z^k\right)\neq 0, \tag{6}$$

where, for convenience,

$$b_k=\begin{pmatrix}\mu\\ k\end{pmatrix} \text{ and } c_k=\begin{pmatrix}\nu\\ k\end{pmatrix}.$$

Considering the Cauchy product of the first two factors, (6) can be rewritten as follows:

$$\left(1+\sum_{k=2}^{\infty} B_k z^{k-1}\right)\left(\sum_{k=0}^{\infty} c_k z^k\right)\neq 0, \tag{6}$$

where

$$B_k=\sum_{k=0}^{\infty}(-1)^{k-j} A_k \begin{pmatrix}\mu\\ k-j\end{pmatrix} z^k.$$

Furthermore, by applying the same method for the Cauchy product in (6), we find that

$$1+\sum_{k=2}^{\infty}\left(\sum_{t=1}^{k} B_t \begin{pmatrix}\nu\\ k-t\end{pmatrix}\right) z^{k-1}\neq 0,$$

or, equivalently, that

$$1+\sum_{k=2}^{\infty}\left(\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j} A_k \begin{pmatrix}\mu\\ t-j\end{pmatrix}\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right) z^{k-1}\neq 0.$$

Thus, if $f(z)\in A$ satisfies the following inequality:

$$\sum_{k=2}^{\infty}\left|\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j} A_k \begin{pmatrix}\mu\\ t-j\end{pmatrix}\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right|\leq 1.$$

Then

$$\sum_{k=2}^{\infty}\left|\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j}\frac{(\psi+1)j^{\alpha}}{2(1-\gamma)}\left(\frac{1+q+\lambda[\beta+j-2]}{1+q+\lambda(\beta-1)}\right)^{m} c(\delta,j)\begin{pmatrix}\mu\\ t-j\end{pmatrix} a_j(\beta)\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right|\leq 1,$$

that is, if

$$\frac{1}{2(1-\gamma)(1+q+\lambda(\beta-1))}\sum_{k=2}^{\infty}|\sum_{t=1}^{k}(\sum_{j=1}^{t}(-1)^{t-j}\, j^{\alpha}\left(1+q+\lambda[\beta+j-2]\right)^{m} c(\delta,j)$$
$$\begin{pmatrix}\mu\\ t-j\end{pmatrix}(\psi+1)a_j(\beta))\begin{pmatrix}\nu\\ k-t\end{pmatrix}|$$

$$\leq\frac{1}{2(1-\gamma)(1+q+\lambda(\beta-1))}$$
$$\left(\left|\sum_{k=2}^{\infty}|\sum_{t=1}^{k}(\sum_{j=1}^{t}(-1)^{t-j}\, j^{\alpha}\left(1+q+\lambda[\beta+j-2]\right)^{m} c(\delta,j)\begin{pmatrix}\mu\\ t-j\end{pmatrix}a_j(\beta))\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right|\right)$$

$$+\frac{1}{2(1-\gamma)(1+q+\lambda(\beta-1))}$$
$$\left(|\psi||\sum_{t=1}^{k}(\sum_{j=1}^{t}(-1)^{t-j}\, j^{\alpha}\left(1+q+\lambda[\beta+j-2]\right)^{m} c(\delta,j)\begin{pmatrix}\mu\\ t-j\end{pmatrix}a_j(\beta))\begin{pmatrix}\nu\\ k-t\end{pmatrix}|\right)\leq 1$$

$$\leq\frac{1}{(1-\gamma)(1+q+\lambda(\beta-1))}\sum_{k=2}^{\infty}|\sum_{t=1}^{k}(\sum_{j=1}^{t}(-1)^{t-j}\, j^{\alpha}\left(1+q+\lambda[\beta+j-2]\right)^{m} c(\delta,j)$$
$$\begin{pmatrix}\mu\\ t-j\end{pmatrix}a_j(\beta))\begin{pmatrix}\nu\\ k-t\end{pmatrix}|\leq 1,$$

Then $f(z)\in T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$. This completes the proof of Theorem 2.4.

Setting $q=l, m=n, \alpha=\delta=0$ theorem 2.4, we get result in [6].

Corollary 2.1 *If $f(z)\in A$ satisfies the following condition:*

$$\sum_{k=2}^{\infty}\left(\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j}(1+\lambda(\beta+j-2)+l)^{n}\begin{pmatrix}\mu\\ t-j\end{pmatrix}a_j(\beta)\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right)$$
$$\leq(1-\gamma)(1+\lambda(\beta-1)+l),$$

then $f(z)\in T_n^{\alpha}(l,\lambda,\beta)$.

Setting $q=0, m=\gamma=\alpha=\delta=0, \beta=1$ Theorem 2.4,we get result in [5].

Corollary 2.2 *If* $f(z)\in A$ *satisfies the following condition:*

$$\sum_{k=2}^{\infty}\left(\sum_{t=1}^{k}\left[\sum_{j=1}^{t}(-1)^{t-j}\begin{pmatrix}\mu\\ t-j\end{pmatrix}a_j\right]\begin{pmatrix}\nu\\ k-t\end{pmatrix}\right)$$

$$\le(1-\gamma),$$

then $f(z)\in T(\gamma)$.

3 Coefficient bounds for functions in the subclass $T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$

In this section, we consider the coefficient bound for functions $f\in T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$, and all the parameters remain as initially defined.

Theorem 3.1 *If* $T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$, *then*

$$|a_2|\le\frac{2(1-\gamma)}{\beta\psi_1^m},$$

$$|a_3|\le\begin{cases}\dfrac{2(1-\gamma)}{\beta\psi_2^m}-\dfrac{2(\beta-1)(1-\gamma)^2}{(\beta)^2(\psi_1^m)^2} & \text{if}\quad 0<\beta<1,\\[2ex] \dfrac{2(1-\gamma)}{\beta\psi_2^m} & \text{if}\quad \beta\ge1.\end{cases}$$

$$|a_4|\le\begin{cases}\dfrac{2(1-\gamma)}{\beta\psi_2^m}-\dfrac{4(\beta-1)(1-\gamma)^2}{(\beta)^2\psi_2^m\psi_1^m}-\dfrac{4(\beta-1)^2(1-\gamma)^3}{(\beta)^3\psi_1^{3m}} & \text{if}\quad 0<\beta<1,\\[2ex] \dfrac{2(1-\gamma)}{\beta\psi_2^m}-\dfrac{4(\beta-1)^2(1-\gamma)^3}{(\beta)^3\psi_1^{3m}}-\dfrac{4(\beta-1)(\beta-2)(1-\gamma)^3}{3(\beta)^3\psi_1^{3m}} & \text{if}\quad 1\le\beta<2,\\[2ex] \dfrac{2(1-\gamma)}{\beta\psi_2^m}-\dfrac{4(\beta-1)(\beta-2)(1-\gamma)^3}{3(\beta)^3\psi_1^{3m}} & \text{if}\quad 2\le\beta<\infty,\end{cases}$$

where

$$\psi_1^m=\left(\frac{1+q+\lambda\beta}{1+q+\lambda(\beta-1)}\right)^m k^{\alpha}c(\delta,k),$$

and

$$\psi_2^m = \left(\frac{1+q+\lambda(\beta+1)}{1+q+\lambda(\beta-1)}\right)^m k^{\alpha} c(\delta,k).$$

Proof: Note that, for $f \in T^{\alpha,\delta}(m,q,\lambda,\beta,\gamma)$

$$\Re\left\{\frac{D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)}{(\frac{1+\lambda(\beta-1)+l}{1+q})^m z^{\beta}}\right\} > \gamma, \quad z \in \mathrm{U}.$$

If we defined the function $P(z)$ by

$$\Re\left\{\frac{\frac{D^{\alpha,\delta}(m,q,\lambda)f^{\beta}(z)}{(\frac{1+\lambda(\beta-1)+l}{1+q})^m z^{\beta}} - \gamma}{1-\gamma}\right\} = 1 + c_1(z) + c_2(z) + \cdots.$$

Then $p(z)$ is analytic in U with $p(0)=1$ and $\Re p(z) > 0$, $z \in \mathrm{U}$. For the clarity we let

$$\left(f(z)\right)^{\beta} = z^{\beta}\left(1+\sum_{j=1}^{\infty}\beta_j (a_1 z + a_2 z^2 + \ldots)^j\right)^{\beta}, \tag{7}$$

where for convenience in the above we let

$$\beta_j = \begin{pmatrix} \beta \\ j \\ \end{pmatrix} \quad j = 1,2,3\cdots, \tag{8}$$

hence from (7) and (8) we have

$$p(z) = 1 + \frac{1}{1-\gamma}(\beta a_2)\left(\frac{1+q+\lambda\beta}{1+q+\lambda(\beta-1)}\right)^m k^{\alpha} c(\delta,k) z + \frac{1}{1-\gamma}(\beta a_3 - \frac{\beta(\beta-1)a_2^2}{2!})$$

$$\left(\frac{1+q+\lambda(\beta+1)}{1+q+\lambda(\beta-1)}\right)^m k^{\alpha} c(\delta,k) z^2 + \frac{1}{1-\gamma}(\beta a_4 - \beta(\beta-1)a_2 a_3 + \frac{(\beta-1)(\beta-2)a_2^3}{3!}) z^3$$

$$\left(\frac{1+q+\lambda(\beta+2)}{1+q+\lambda(\beta-1)}\right)^m k^{\alpha} c(\delta,k) z^4 + \cdots. \tag{9}$$

On comparing coefficients in (9) and using the fact that the $|c_k| \le 2, k \le 1$,
the results follow and the proof is complete.

setting $\lambda = 1$, $\alpha,\delta = 0$ and $q = 0$ in the Theorem3.1, we get the result in [6].

Corollary 3.2 *If* $T^{0,0}(m,0,1,\gamma) = T_n^{\beta}(\gamma)$, *then*

$$|a_2| \leq \frac{2(1-\gamma)\beta^{m-1}}{(1+\beta)^m},$$

$$|a_3| \leq \begin{cases} \dfrac{2(1-\gamma)\beta^{m-1}}{(\beta+2)^m} - \dfrac{2(\beta-1)(1-\gamma)^2\beta^{2m-2}}{(1+\beta)^{2m}} & if \quad 0<\beta<1, \\ \\ \dfrac{2(1-\gamma)\beta^{m-1}}{(\beta+2)^m} & if \quad \beta \geq 1. \end{cases}$$

$$|a_4| \leq \begin{cases} \dfrac{2(1-\gamma)\beta^{m-1}}{(\beta+2)^m} - \dfrac{4(\beta-1)\beta^{2m-2}(1-\gamma)^2}{(\beta+1)^m(\beta+2)^m} - \dfrac{4(\beta-1)^2\beta^{3m-3}(1-\gamma)^3}{(1+\beta)^3} & if \quad 0<\beta<1, \\ \\ \dfrac{2(1-\gamma)\beta^{m-1}}{(\beta+2)^m} - \dfrac{4(\beta-1)^2\beta^{3m-3}(1-\gamma)^3}{(1+\beta)^{3m}} - \dfrac{4(\beta-2)(\beta-1)\beta^{3m-3}(1-\gamma)^3}{3(\beta+1)^{3m}} & if \quad 1\leq\beta<2, \\ \\ \dfrac{2(1-\gamma)\beta^{m-1}}{(\beta+2)^m} - \dfrac{4(\beta-1)(\beta-2)\beta 3m-3(1-\gamma)^3}{3(\beta+1)^{3m}} & if \quad 2\leq\beta<\infty. \end{cases}$$

4. Conclusion

Finaly, in this study the researchers showed and proved some propiereties for a new subclass of Bazilevic functions defined by a generalized derivative operator $D^{\alpha,\delta}(m,q,\lambda)$..

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