ISSN: 2706-9087

ISSUE 8 DESEMBER 2019

مجلة العلوم الإنسانية والتطبيقية
Journal of Humanitarian and Applied Sciences

العدد 8 ديسمبر 2019

A Comparison Between Elzaki and Aboodh Transforms to Solve Systems of Linear Volterra Integral Equations

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الملخص

في هذه الورقة العلمية ، أجريت مقارنة بين تحويل الزاكي و تحويل عبود لحل بعض منظومات معادلات فولتيرا التكاملية الخطية.

Abstract

In this paper , the Elzaki transform is compared with Aboodh transform to solve some systems of linear Volterra integral equations.

Keywords: Elzaki transform;Aboodh transform ; systems of Linear Volterra integral equations.

Introduction

Integral transforms play an important role in many fields of science. In literature, integral transforms are widely used in mathematical physics, optics, engineering mathematics and, few others. Among these transforms which were extensively used and applied on theory and applications are: Laplace transform , Fourier , Mellin, Hankel , Mahgoub , Elzaki , Aboodh and Sumudu.

The Laplace transform has been effectively used to solve linear and non-linear ordinary and partial differential equations and integral equations [15] and is used extensively in electrical engineering. The Laplace transform reduces a linear differential equation to an algebraic equation, which can be solved by rules of algebra. The original differential equation can then be solved by applying the inverse Laplace transform.
Tarig M. Elzaki and Salih M. Elzaki in 2011, showed the modified of Sumudu transform [6-13] or Elzaki transform was applied to partial differential equations, ordinary differential equations, system of ordinary and partial differential equations and integral equations. Elzaki transform is a powerful tool for solving some differential equations which cannot be solved by Sumudu transform.

Aboodh Transform [1-5,14] was introduced by Khalid Aboodh in 2013, to facilitate the process of solving ordinary and partial differential equations in the time domain. This transformation has deeper connection with the Laplace and Elzaki Transform.

The main objective in this article is to introduce a comparative study to solve some systems of linear Volterra integral equations by using Elzaki transform and Aboodh transform.

Definitions and Standard Results

1. Elzaki Transform

The Elzaki transform is a newly introduced integral transform similar to Laplace transform and other integral transform that are defined in the time domain $t \geq 0$, and for functions in the set B defined by:

$$B = \left\{ f \left| \exists M, k_1, k_2 > 0 \text{ such that} \left| f(t) \right| < M e^{\frac{|t|}{k_j}}, if\ t \in \left(-1\right)^j \times [0,\infty) \right. \right\} \tag{1}$$

For a given function in the set B , the constant M must be finite number ; k_1, k_2 may be finite or infinite.

Then, the Elzaki transform denoted by the operator $E[f(t)] = T(u)$ for function of exponential order and belonging to set B is defined by one of the following equivalent integral forms:

$$\begin{aligned} T\left(u\right) &= E[f\left(t\right)] = u\int_0^\infty e^{\frac{-t}{u}} f\left(t\right) dt, u \in \left(k_1, k_2\right) \\ T(u) &= E[f(t)] = u^2 \int_0^\infty f(ut) e^{-t} dt, k_1, k_2 > 0 \end{aligned} \tag{2}$$

1.1 Some Properties of Elzaki Transform

1.1.1 Linearity Property of Elzaki Transform

` If $E\left[f_1\left(t\right)\right] = T_1\left(u\right)$ and $E\left[f_2\left(t\right)\right] = T_2\left(u\right)$ then

$$E\left[a\ f_1\left(t\right) + b\ f_2\left(t\right)\right] = a\,E\left[f_1\left(t\right)\right] + b\,E\left[f_2\left(t\right)\right] = a\,T_1\left(u\right) + b\ T_2\left(u\right) \tag{3}$$

Where a, b arbitrary constants

1.1.2 Convolution of Two Functions

Convolution of two functions $f(t)$, $h(t)$ is denoted by $f(t) * h(t)$ and it is defined by

$$f(t) * h(t) = h(t) * f(t) = \int_0^t f(x)h(t-x)dx = \int_0^t h(x)f(t-x)dx \tag{4}$$

If $E[f(t)] = T(u)$, $E[h(t)] = W(u)$ then

$$E[f(t) * h(t)] = \frac{1}{u} E[f(t)]E[h(t)] = \frac{1}{u} T(u)W(u) \tag{5}$$

1.1.3 Elzaki Transforms of the derivatives

If $E[f(t)] = T(u)$ then

1. $$E[f'(t)] = \frac{1}{u} T(u) - u\,f(0) \tag{6}$$

2. $$E[f''(t)] = \frac{1}{u^2}T(u) - f(0) - u\,f'(0) \tag{7}$$

3. $$E[f^{(m)}(t)] = u^{-m}T(u) - \sum_{k=0}^{m-1} u^{2-m+k} f^{(k)}(0) \tag{8}$$

1.1.4 Multiple Shift Property

If $E[f(t)] = T(u)$ then

1. $$E[t\,f(t)] = u^2 \frac{d}{du}[T(u)] - u\,T(u) \tag{9}$$

2. $$E[t^2\,f(t)] = u^4 \frac{d^2}{du^2}[T(u)] \tag{10}$$

3. $$E[t^3 f(t)] = u^6 \frac{d^3}{du^3}[T(u)] + 3u^5 \frac{d^2}{du^2}T(u) \tag{11}$$

4. $$E[t\,f'(t)] = u^2 \frac{d}{du}[\frac{1}{u}T(u) \text{ - } u\,f(0)] \text{ - } u[\frac{T(u)}{u} \text{ - } u\,f(0)]$$ (12)

5. $$E[t^2\,f'(t)] = u^4 \frac{d^2}{du^2}\left[\frac{1}{u}T(u) - u\,f(0)\right] \tag{13}$$

6. $E[t\,f''(t)] =$
$$u^2 \frac{d}{du}\left[\frac{T(u)}{u^2} - f(0) - u\,f'(0)\right] - u\,[\frac{T(u)}{u^2} - f(0) - u\,f'(0)] \tag{14}$$

7. $$E[t^2\,f''(t)] = u^4 \frac{d^2}{du^2}\left[\frac{1}{u^2}T(u) - f(0) - u\,f'(0)\right] \tag{15}$$

1.2 Elzaki Transform of Some Elementary Function

S.N.	$f(t)$	$E[f(t)]$
1.	1	u^2
2.	t	u^3
3.	$t^n, n \geq 0$	$n!u^{n+2}$
4.	e^{at}	$\frac{u^2}{1-au}$
5.	$t\,e^{-at}$	$\frac{u^3}{(1+au)^2}$
6.	$\sin at$	$\frac{au^3}{1+a^2u^2}$
7.	$\cos at$	$\frac{u^2}{1+a^2u^2}$
8.	$\sinh at$	$\frac{au^3}{1-a^2u^2}$
9.	$\cosh at$	$\frac{au^2}{1-a^2u^2}$
10.	$J_0(t)$	$\frac{u^2}{\sqrt{1+u^2}}$

NB. In the previous table a is a constant

2. Aboodh Transform

Aboodh transform is a new integral transform defined for function of exponential order .Consider functions in the set A , defined by:

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, \left| f(t) \right| < Me^{-vt} \right\}, k_1 \leq v \leq k_2 \qquad (16)$$

For a given function in the set A, M must be a finite number, k_1, k_2 may be finite or infinite. Aboodh transform which is defined by the integral equation

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}\,dt, t \geq 0, k_1 \leq v \leq k_2 \qquad (17)$$

2.1 Some Properties of Aboodh Transform

2.1.1 Linearity Property of Aboodh Transform

` If $A\left[f_1(t)\right] = K_1(v)$ and $A\left[f_2(t)\right] = K_2(v)$ then

$$A\left[a\, f_1(t) + b\, f_2(t)\right] = a\,A\left[f_1(t)\right] + b\,A\left[f_2(t)\right] = a\,K_1(v) + b\,K_2(v) \qquad (18)$$

ISSN: 2706-9087

ISSUE 8 DESEMBER 2019

مجلة العلوم الإنسانية والتطبيقية
Journal of Humanitarian and Applied Sciences

العدد 8 ديسمبر 2019

Where a, b arbitrary constants

2.1.2 Convolution of Two Function

Convolution of two functions $f(t)$, $h(t)$ is denoted by $f(t)*h(t)$ and it is defined by

$$f(t)*h(t)=h(t)*f(t)=\int_0^t f(x)h(t-x)dx=\int_0^t h(x)f(t-x)dx \qquad (19)$$

If $A[f(t)]=K(v)$, $A[h(t)]=L(v)$ then

$$A[f(t)*h(t)]=v\,A[f(t)]\;A[h(t)]=vK(v)\,L(v) \qquad (20)$$

2.1.3 Aboodh Transform of the Derivatives

If $A[f(t)]=T(v)$ then

1. $A[f'(t)]=v\,K(v)-\dfrac{f(0)}{v}$ (21)
2. $A[f''(t)]=v^2K(v)-f(0)-\dfrac{f'(0)}{v}$ (22)
3. $A[f^{(m)}(t)]=v^mK(v)-\displaystyle\sum_{k=0}^{m-1}\frac{f^{(k)}(0)}{u^{2-m+k}}$ (23)

2.1.4 Multiple Shift Property

If $A[f(t)]=K(v)$ then

1. $A[t\,f(t)]=-\dfrac{d}{dv}[K(v)]-\dfrac{1}{v}K(v)$ (24)
2. $A[t\,f'(t)]=-\dfrac{d}{dv}[v\,K(v)-\dfrac{f(0)}{v}]-\dfrac{1}{v}\left[vK(v)-\dfrac{f(0)}{v}\right]$ (25)
3. $A[t\,f''(t)]=\dfrac{-d}{dv}\left[\dfrac{K(v)}{v^{-2}}-\dfrac{f'(0)}{v}-f(0)\right]-\dfrac{1}{v}\left[\dfrac{K(v)}{v^{-2}}-\dfrac{f'(0)}{v}-f(0)\right]$ (26)
4. $A\left[t^2f'(t)\right]=v\dfrac{d^2K(v)}{dv^2}+2\dfrac{dK(v)}{dv}-2\dfrac{f(0)}{v^3}$ (27)
5. $A\left[t^2f''(t)\right]=v^2\dfrac{d^2K(v)}{dv^2}+4v\dfrac{dK(v)}{dv}+2K(v)-2\dfrac{f'(0)}{v^3}$ (28)

2.1.5 Aboodh Transform of Some Elementary Functions

$S.N.$	$f(t)$	$A\left[f(t)\right]$
1.	1	$\dfrac{1}{v^2}$

ISSN: 2706-9087

ISSUE 8 DESEMBER 2019

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2.	t	$\frac{1}{v^3}$
3.	$t^n, n \geq 0$	$\frac{n!}{v^{n+2}}$
4.	e^{at}	$\frac{1}{v^2 - av}$
5.	$t\,e^{-at}$	$\frac{1}{v\left(v+a\right)^2}$
6.	$\sin at$	$\frac{a}{v(v^2+a^2)}$
7.	$\cos at$	$\frac{1}{(v^2+a^2)}$
8.	$\sinh at$	$\frac{a}{v(v^2-a^2)}$
9.	$\cosh at$	$\frac{1}{(v^2-a^2)}$
10.	$J_0(t)$	$\frac{1}{v\sqrt{\left(1+v^2\right)}}$

NB. In the previous table a is a constant

Applications

Example 1. Solve the system of Volterra integral equations

$$\begin{cases} f(t) = 2 - e^{-t} + \int_0^t \left[\left(t-x\right)f(x) + \left(t-x\right)g(x)\right]dx \\ g(t) = 2t - e^{t} + 2e^{-t} + \int_0^t \left[\left(t-x\right)f(x) - \left(t-x\right)g(x)\right]dx \end{cases} \tag{29}$$

Solution:

1. Taking Elzaki transform of both sides of each equation in (29) gives

$$\begin{cases} E[f][1-u^2] - u^2\,E[g] = \dfrac{u^2+2u^3}{1+u} \\ E[g][1+u^2] - u^2\,E[f] = \dfrac{u^2-u^3-2u^5}{1-u^2} \end{cases} \tag{30}$$

ISSN: 2706-9087

ISSUE 8 DESEMBER 2019

مجلة العلوم الإنسانية والتطبيقية
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Solving this system of equations for $E[f]$, $E[g]$ gives

$$\begin{cases} E[f]=\dfrac{u^2}{1-u} \\ E[g]=\dfrac{u^2}{1+u} \end{cases} \tag{31}$$

By taking the inverse Elzaki transform of both sides of each equation in (31) , the exact solutions are given by

$$\left(f(t),g(t)\right)=\left(e^t,e^{-t}\right) \tag{32}$$

2. Taking Aboodh transform of both sides of each equation in (29) gives

$$\begin{cases} A[f]\,[v^2-1]-A[g]=\dfrac{2+v}{1+v} \\ A[g][1+v^2]-A[f]=\dfrac{-2-v^2+v^3}{v(v^2-1)} \end{cases} \tag{33}$$

Solving this system of equations for $A[f]$, $A[g]$ gives

$$\begin{cases} A[f]=\dfrac{1}{v^2-v} \\ A[g]=\dfrac{1}{v^2+v} \end{cases} \tag{34}$$

By taking the inverse Aboodh transform of both sides of each equation in (34) , the exact solutions are given by

$$\left(f(t),g(t)\right)=\left(e^t,e^{-t}\right) \tag{35}$$

Example 2. Solve the system of Volterra integral equations

$$\begin{cases} \dfrac{1}{2}t^2+\dfrac{1}{2}t^3+\dfrac{1}{12}t^4=\displaystyle\int_0^t\left[\left(t-x-1\right)f(x)+\left(t-x+1\right)g(x)\right]dx \\ \dfrac{3}{2}t^2-\dfrac{1}{6}t^3+\dfrac{1}{12}t^4=\displaystyle\int_0^t\left[\left(t-x+1\right)f(x)+\left(t-x-1\right)g(x)\right]dx \end{cases} \tag{36}$$

Solution:

1. Taking Elzaki transform of both sides of each equation in (36) gives

$$\begin{cases} u^3+3u^4+2u^5=[u-1]E[f]+[u+1]E[g] \\ 3u^3-u^4+2u^5=[u+1]E[f]+[u-1]E[g] \end{cases} \tag{37}$$

Solving this system of equations for $E[f]$, $E[g]$ gives

ISSN: 2706-9087

ISSUE 8 DESEMBER 2019

مجلة العلوم الإنسانية والتطبيقية
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$$\begin{cases} E[f]=u^2+u^3 \\ E[g]=u^2+2u^4 \end{cases} \tag{38}$$

By taking the inverse Elzaki transform of both sides of each equation in (38) , the exact solutions are given by

$$\left(f(t),g(t)\right)=\left(1+t,1+t^2\right) \tag{39}$$

2. Taking Aboodh transform of both sides of each equation in (36) gives

$$\begin{cases} 2+3v+v^2=\left(v^4-v^5\right)A[f]+\left(v^4+v^5\right)A[g] \\ 2-v+3v^2=\left(v^4+v^5\right)A[f]+\left(v^4-v^5\right)A[g] \end{cases} \tag{40}$$

Solving this system of equations for $A[f]$, $A[g]$ gives

$$\begin{cases} A[f]=\dfrac{1}{v^3}+\dfrac{1}{v^2} \\ A[g]=\dfrac{1}{v^2}+\dfrac{2}{v^4} \end{cases} \tag{41}$$

By taking the inverse Aboodh transform of both sides of each equation in (41) , the exact solutions are given by

$$\left(f(t),g(t)\right)=\left(1+t,1+t^2\right) \tag{42}$$

Example 3. Solve the system of Volterra integral equations

$$\begin{cases} f(t)=t-\dfrac{1}{12}t^4-\dfrac{1}{20}t^5+\displaystyle\int_0^t\left[\left(t-x\right)g(x)+\left(t-x\right)h(x)\right]dx \\ g(t)=t^2-\dfrac{1}{6}t^3-\dfrac{1}{20}t^5+\displaystyle\int_0^t\left[\left(t-x\right)f(x)+\left(t-x\right)h(x)\right]dx \\ h(t)=\dfrac{5}{6}t^3-\dfrac{1}{12}t^4+\displaystyle\int_0^t\left[\left(t-x\right)f(x)+\left(t-x\right)g(x)\right]dx \end{cases} \tag{43}$$

Solution:

1. Taking Elzaki transform of both sides of each equation in (43) gives

$$\begin{cases} E[f]=u^3-2u^6-6u^7+u^2E[g]+u^2E[h] \\ E[g]=2u^4-u^5-6u^7+u^2E[f]+u^2E[h] \\ E[h]=5u^5-2u^6+u^2E[f]+u^2E[g] \end{cases} \tag{44}$$

Solving this system of equations for $E[f]$, $E[g]$, $E[h]$ gives

$$\begin{cases} E[f]=u^3 \\ E[g]=2u^4 \\ E[h]=6u^5 \end{cases} \tag{45}$$

By taking the inverse Elzaki transform of both sides of each equation in (45) , the exact solutions are given by

$$\left(f(t),g(t),h(t)\right)=\left(t,t^2,t^3\right) \tag{46}$$

2. Taking Aboodh transform of both sides of each equation in (43) gives

$$\begin{cases} v^7\, A[f]=v^4-2v-6+v^5A[g]+v^5A[h] \\ v^7\, A[g]=2v^3-v^2-6+v^5A[f]+v^5A[h] \\ v^7\, A[h]=5v^2-2v+v^5A[f]+v^5A[g] \end{cases} \tag{47}$$

Solving this system of equations for $A[f]$, $A[g]$, $A[h]$ gives

$$\begin{cases} A[f]=\dfrac{1}{v^3} \\ A[g]=\dfrac{2}{v^4} \\ A[h]=\dfrac{6}{v^5} \end{cases} \tag{48}$$

By taking the inverse Aboodh transform of both sides of each equation in (48) , the exact solutions are given by

$$\left(f(t),g(t),h(t)\right)=\left(t,t^2,t^3\right) \tag{49}$$

Example 4. Solve the system of Volterra integral equations

$$\begin{cases} -2+2\cosh t=\displaystyle\int_0^t\left[f(x)-g(x)\right]dx \\ -1+t+\dfrac{1}{2}t^2+e^t=\displaystyle\int_0^t\left[\left(t-x+1\right)g(x)-\left(t-x-1\right)h(x)\right]dx \\ t+\dfrac{1}{2}t^2+t\,e^t=\displaystyle\int_0^t\left[\left(t-x\right)h(x)+\left(t-x+1\right)f(x)\right]dx \end{cases} \tag{50}$$

Solution:

3. Taking Elzaki transform of both sides of each equation in (50) gives

ISSN: 2706-9087
ISSUE 8 DESEMBER 2019
مجلة العلوم الإنسانية والتطبيقية
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العدد 8 ديسمبر 2019

$$\begin{cases} \dfrac{2u^4}{1-u^2} = u\,E[f] + uE[g] \\ \dfrac{2u^2-u^4}{1-u} = \left[u+1\right]E[g] - \left[u-1\right]E[h] \\ \dfrac{2u^2-u^3-u^4+u^5}{(u-1)^2} = uE[h] + \left[u+1\right]E[f] \end{cases} \tag{51}$$

Solving this system of equations for $E[f]$, $E[g]$, $E[h]$ gives

$$\begin{cases} E[f] = \dfrac{u^2}{1-u} + u^2 \\ E[g] = u^2 + \dfrac{u^2}{1+u} \\ E[h] = \dfrac{u^3}{\left(1-u\right)^2} \end{cases} \tag{52}$$

By taking the inverse Elzaki transform of both sides of each equation in (52) , the exact solutions are given by

$$\left(f(t), g(t), h(t)\right) = \left(1+e^t, 1+e^{-t}, t\,e^t\right) \tag{53}$$

4. Taking Aboodh transform of both sides of each equation in (50) gives

$$\begin{cases} \dfrac{2}{v\left(v^2-1\right)} = A[f] - A[g] \\ \dfrac{2v^2-1}{v^2\left(v-1\right)} = \left(1+v\right)A[g] + \left(v-1\right)A[h] \\ \dfrac{1-v-v^2+2v^3}{v^2\left(v-1\right)^2} = \left(1+v\right)A[f] + A[h] \end{cases} \tag{54}$$

Solving this system of equations for $A[f]$, $A[g]$, $A[h]$ gives

$$\begin{cases} A[f] = \dfrac{1}{v^2} + \dfrac{1}{v^2-v} \\ A[g] = \dfrac{1}{v^2} + \dfrac{1}{v^2+v} \\ A[h] = \dfrac{1}{v\left(v-1\right)^2} \end{cases} \tag{55}$$

By taking the inverse Aboodh transform of both sides of each equation in (55) , the exact solutions are given by

$$\left(f(t), g(t), h(t) \right) = \left(1+e^{t}, 1+e^{-t}, t\,e^{t} \right) \tag{56}$$

Conclusion

In this paper, we have successfully discussed the comparative study of Elzaki and Aboodh transforms in solving systems of linear Volterra integral equations. The given applications in applications section show that both transforms (Elzaki and Aboodh transforms) are closely connected to each other, and they are both powerful and efficient methods. Finally, all solutions obtained in this article have been checked by putting them back into the original system.

References

[1] Aboodh, K.S. (2013).*The new integral transform "Aboodh transform"* ,Global Journal of Pure and Applied Mathematics, **9** (1), 35-43

[2] Aboodh, K. S. (2014). *Application of new transform "Aboodh transform" to partial differential equations*, Global Journal of Pure and Applied Math, **10** (2), 249-254

[3] Aggarwal, S. and Chauhan, R. (2019). *A comparative study of Mohand and Aboodh transforms*, International Journal of Research in Advent Technology, **7**(1), 520-529.

[4] Aggrawal, S. ,Sharma,N. and Chauhan, R. , *Application of Aboodh transform for solving linear Volterra Integro-differential Equations of Second Kind* , International Journal of Research in Advent Technology ,**6**(6) ,1186-1190.

[5] Alshikh ,A. , and Mahgoub ,M. (2016). *Solving systems of ordinary differential equations by Aboodh transform* , **34**(9).1144-1148.

[6] ELtayeb, H. and Kilicman, A. (2010), *On some applications of a new integral transform*, International Journal of Mathematical Analysis, **4**(3), 123-132.

[7] Eltayeb , H. and Kilicman,A. (2010), *A note on the Sumudu transforms and differential Equations*, Applied Mathematical Sciences, **4**(22), 1089-1098.

[8] Elzaki,T. M. (2011). *The new integral transform "Elzaki transform"*, Global Journal of Pure and Applied Mathematics, Number 1, 57-64.

[9] Elzaki T.M. & Elzaki, S.M. (2011), *Application of new transform "Elzaki transform" to partial differential equations*, Global Journal of Pure and Applied Mathematics, **7**(1), 65-70.

[10] Elzaki T.M. & Elzaki, S.M. (2011), *On the Connections between Laplace and Elzaki transforms*, Advances in Theoretical and Applied Mathematics, **6** (1), 1-11.

[11] Elzaki T.M. & Elzaki, S.M. (2011), *On the Elzaki transform and ordinary differential equation with variable coefficients*, Advances in Theoretical and Applied Mathematics. **6**(1), 13-18.

[12] Kilicman A. and ELtayeb, H. (2010), *A note on integral transform and partial differential equation*, Applied Mathematical Sciences, **4** (3), 109-118.

ISSN: 2706-9087

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[13] Kilicman, A. and Gadain, H.E. (2009), *An application of double Laplace transform and Sumudu transform*, Lobachevskii J. Math. **30** (3), 214-223.

[14] Mahgoub ,M. ,Aboodh, K., and Alshikh , A.(2016). *On the solution of ordinary differential equation with variable coefficients using Aboodh transform*, Advances in Theoretical and Applied Mathematics, **11**(4) ,383-389.

[15] Wazwaz, A.M. ,(2011). *Linear and nonlinear integral equations* , Higher Education Press, Beijing.